Successive Localization of Mobile Sensor Nodes Using Non-Parametric Belief Propagation

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Abstract: In this article, we propose a method for positioning networked sensor nodes that include mobile ones. When nodes can measure the relative distances to their neighbor ones, we can estimate the absolute position of each node. Once distributed static nodes are positioned, we can position a mobile node for each time that moves through the distributed ones and measures the relative distances to the distributed ones. By using these distances, we can improve the estimated positions of the distributed nodes. For this estimation, we employ the non-parametric belief propagation. Simulation results showed that the proposed method improved the estimated positions.

Keywords: sensor network, sensor localization, non-parametric belief propagation, mobile node

1. INTRODUCTION

When we collect measurements obtained by distributed sensor nodes, we need the information on the position of each node. Without the information, we cannot know the spatial distribution of the measurements. For estimating the absolute position, the GPS is widely used. If each sensor node has a GPS receiver and it detects enough signals from the GPS satellites, the node can accurately estimate its absolute position. In other cases, each sensor node should estimate its absolute position without using the GPS.

Many methods have been proposed for estimating the absolute positions of sensor nodes. Many of them use measurements of relative distances between sensor nodes. Those methods assume that each sensor can measure the relative distances to its neighboring nodes by referring to, e.g. the radio field intensity for communication. Based on the triangulation, we can estimate the absolute position of each node using a set of those measurements. For such estimation, non-parametric belief propagation is widely used[1].

The belief propagation is an iterative algorithm that estimates the probability distribution on a graphical model, which can represent the independencies of the estimates of the positions. If the relative distance between two sensor nodes is measured and is used for the estimation, the resultant estimates of the positions of the two nodes are dependent on each other. In this case, two nodes in the graphical model correspond to the two estimates, and these nodes are linked together with an edge because they are dependent on.

If each node manages the estimate of its position and measures the relative distances to its neighbors, the node can compute the marginal distributions of the positions of the neighbors. Let d_{ij} denote the relative distance between two nodes i and j and let $p(x_i)$ denote the distribution of the node i. In addition, let C_d denote a circumference of a circle whose center is at the origin and whose radius is equal to d. Then, the node i can estimate the marginal distribution of the node j as $p(x_j) = p(x_i) * C_{d_{ij}}$, where $*$ denotes the convolution. In the framework of the non-parametric distribution, each node computes the marginal distribution of the position of each neighbor node and sends the distribution to the corresponding neighbor as a message. When a node receives the messages from its neighbors, the node integrates these messages and updates the estimate of its position. Iterating these processes, all sensor nodes would obtain the marginal distributions of their positions.

Using the non-parametric belief propagation, we can position static sensor nodes. Once their positions are estimated, we can estimate the position of a mobile sensor node if it measures the relative distances to the neighboring static nodes. In addition, the measurements obtained by the mobile node can improve the estimated positions of the static sensor nodes. In this article, we propose a method that successively improves the estimated positions of static sensor nodes while mobile nodes measure the relative distances to the neighboring static nodes and estimates its position for every time.

2. A GRAPHICAL MODEL FOR POSITIONING SENSOR NODES

In this article, we assume that each sensor node can measure the relative distance to its neighboring nodes. Let $P(O_{ij}|x_i, x_j)$ denote the conditional probability that the relative distance between sensors i and j is observed:

$$P(O_{ij}|x_i, x_j) = \frac{1}{Z} \exp \left\{ -||x_i - x_j||^2 / R^2 \right\},$$

where $O_{ij} = \{0, 1\}$ is equal to one (zero) if the distance is (not) measured respectively. Let $P(d_{ij}|x_i, x_J)$ denote the conditional probability that the value of the measurement becomes to d_{ij}:

$$P(d_{ij}|x_i, x_j) = \frac{1}{Z} \exp \left\{ -(d_{ij} - |x_i - x_j|)^2) / \sigma^2 \right\},$$

where σ^2 is the variance of the measurement noise. Then, we can model the distribution of the absolute positions of the sensor nodes as follows:

$$P(x_1, x_2, \cdots, x_N, \{d_{ij}\}, \{O_{ij}\}) \\ = \prod P(O_{ij}|x_i, x_j) \prod P(d_{ij}|x_i, x_j) \prod P(x_i). \quad (1)$$

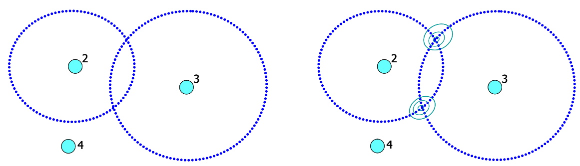
Given a set of measurements $\{O_{ij}\}$, $\{d_{ij}\}$ and x_i for some i and j, we estimate the probability in (1) by means of the non-parametric belief propagation.

3. NON-PARAMETRIC BELIEF PROPAGATION

As described in the previous section, each node sends to its neighboring node a message that represents the merginal distribution. Because it is difficult to parametrically represent the merginal distributions, we represent the distribution with a set of M particles. Each particle has information on its weight w, position m and the variance. If a sensor i measures the relative distance to a node j, the message from sensor i to j is computed as follows:

$$m_{ij}^{(t)} = x_i^{(t)} + (d_{ij} + \nu^{(t)})[\cos\theta^{(t)}, \sin\theta^{(t)}], \quad (2)$$

where (t) denotes the number of particle ($t = 1, 2, \cdots, M$), and ν and θ denote the random variables that obey the normal distribution and the uniform distribution $[0, 2\pi)$. The initial value of the weight $w_i^0(l) = 1/M$.



(A) (B)

Fig. 1 When the 3 nodes distribute as (A) and the node 2 and 3 measure the relative distance to node 1, messages distribute on circumference whose center is node 2 or 3 that send messages. From these messages the position of node 1 estimates the cross of two circumferences as (B). And node 4 also send the information that relative distance between node 1 and 4 doesn't measure, the estimates of node 1 the upper cross in (B).

The message $m_{ij}^{(t)}$ in (2) consists of M particles arranged along a circle whose radius is equal to d_{ij} and center is at x_i (see Fig. 1) Receiving sets of messages from the neighbor nodes, each sensor node computes its marginal distribution of its position by means of the Gibbs sampling method.

Let $|\Gamma_i|$ denote the number of the neighbor nodes that send the message to the node i. For computing the marginal distribution of x_i, the node i firstly selects kM/Γ particles from each message m_{ij} and obtains kM particles in total, where $k < |\Gamma_i|$ is some positive integer determined in advance. The node secondly re-computes the weight w of each particle as follows:

$$w_i^{(l)} = \prod_{v \in \Gamma_i} m_{vi}^{n_s}\left(x_i^{(l)}\right) / \sum_{v \in \Gamma_i^o} m_{vi}^{n_s}\left(x_i^{(l)}\right), \quad (3)$$

where $m_{vi}^{n=s}$ in numerator of equation (3), $v(\in \Gamma_i^O)$ may obtain relative distance between v and i or obtain the information that node v cannot obtain the relative distance between v and i. If node j doesn't obtain the relative distance to node i, message $m_{ij}(x_j)$ is computed as follows:

$$m_{ij}(x_j) = 1 - \sum_k w_i^{(l)} P_o\left(x_j - x_i^{(l)}\right) \quad (4)$$

Finally, the node i selects M particles from the kM particles based on the probability that is proportional to the weight w.

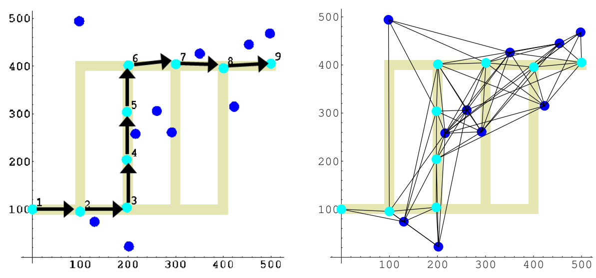
Figure. 1 explain the process of the Gibbs sampling. When the sensor node i receives the messages from two neighboring nodes ($\Gamma = 2$), the node i selects the two circles each of which is represented by the message sent by the corresponding node.

In the proposed method, each node measures the relative distance to it. This information is available to suppress false estimates as shown in Figure. 1.

Iteratively communicating the messages and computing the marginal distributions, we can estimate the probability $P(x_i)$ in (1).

In this article, we consider to distribute a set of static sensor nodes in an environment, each of which can measure the relative distances to its neighboring nodes. The positions of the sensors are estimated by means of the non-parametric belief propagation in advance. Then, a mobile sensor moves through the static nodes. Here, we assume that the mobile sensor node k has two sensors: One sensor is for measuring the relative distances to the neighboring static nodes and the other is for measuring its moving distance. Using these measurements, the mobile sensor node estimates its position at every time by means of the non-parametric belief propagation.

Let $x_k(t)$ denote the position of the mobile sensor k at time t. We consider the mobile node for different times as different nodes in the graphical model (see Fig. 2).



(A) (B)

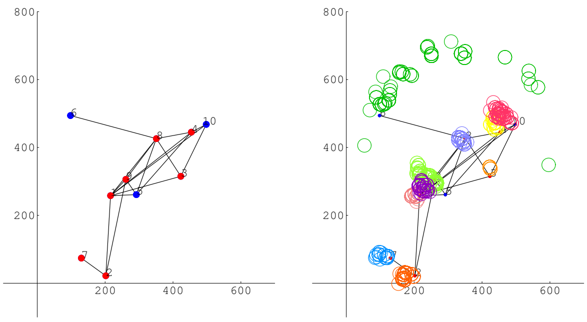
Fig. 2 The mobile node moves through the distributed static node as (A). The position of mobile node at each time regards one node and coressponded graphical model shown as (B).

These nodes that correspond to the mobile sensor of different times are linked by edges because the sensor measures the moving distances. In addition, each of these nodes are linked to other nodes that correspond to the static nodes, because the mobile sensor node measures the relative distances to neighboring static nodes.

Every time the mobile node measures the relative distances to neighboring nodes, the non-parametric belief propagation is applied to estimate the position $x_k(t)$ of the mobile sensor node and to improve the positions of static nodes. Based on this framework, we can successively improve the accuracy of all estimates while the mobile sensor node moves through the environment.

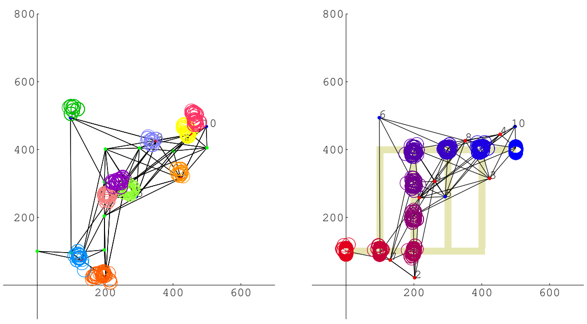
4. EXPERIMENTAL RESULTS

We simulated the proposed positioning method. Figure 3(A) shows the graphical model of static nodes and (B) shows the result of estimates described by particles in the environment that 10 static nodes are distributed and 6 sensor nodes equipping GPS receiver and obtain the absolute position with GPS. As shown in this figure, one sensor node (upper left) has one edge and the estimate of its position has ambiguity, because of triangulation.



(A) (B)

Fig. 3 (A) shows the graphical model of static nodes. (B) shows the estimated position of static nodes.

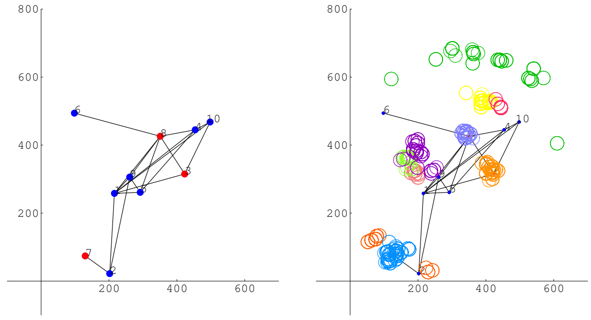


(A) (B)

Fig. 4 (A) shows the estimated position of static nodes by using information of mobile node. (B) shows the estimated position of mobile node at each time.

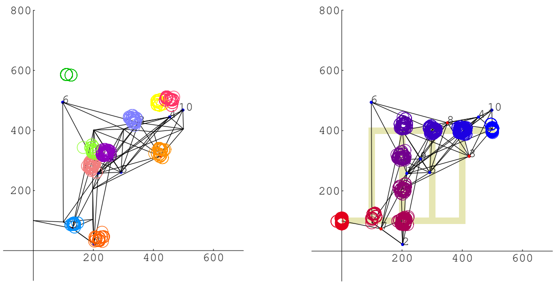
Through the destributed static nodes shown in Figure 3, one mobile node moves along a street as shown in Figure **??**. In Figure 4(A), the entrance is $\{0, [90, 110]\}$ (left in this figure) and the exit is $\{500, [390, 410]\}$ (right in this figure). The mobile sensor node moves from left to right in the figure by constant speed. Figure 4(B) shows the estimated positions of the mobile node for each time. Figure 4(A) shows the improved estimates of the position of the static nodes. Comparing Figure 4(A) with Figure 3(B), we can see that the ambiguity successfully removed.

Next, we shows the result of an other simulation. The simulation assumes tha environment that the only three sensor node has GPS receiver and the absolute position with GPS in advance, because the non-parametric belief propagation is based on triangulation, the estimation need only three absolute position previously. The environment of this simulation that the position of static nodes and mobile one at each time is same at previous simulation, but it is different that the node equipping the GPS receiver is three node(3, 7, 9).



(A) (B)

Fig. 5 (A) shows the graphical model of static nodes. (B) shows the estimated position of static nodes.



(A) (B)

Fig. 6 (A) shows the estimated position of static nodes by using information of mobile node. (B) shows the estimated position of mobile node at each time.

Comparing Figure 5(B) with Figure 3(B), the estimates in the this situation has more ambiguity. But after the mobile node moves, the estimates of static node removes the ambiguity and it is same as the previous simulation that is seen Figure 6. And Figure 6(B) shows the postion of mobile node at each time estimated well.

We shows the graph the trace of covariant matrix that represent the estimate accuracy.

The trace of covariant matrix is calculated as follows;

$$Tr[E] = \sum_{i=1}^{N} \sum_{k=1}^{M} (x_i^k - \overline{x_i})^2, \tag{5}$$

The Figure 7 is the total of variance in static nodes at each time in the environment 10 static nodes distributed including the 6 static nodes has GPS receiver, and the Figure 8 is the covariance in the environment the 10 static nodes distributed including 3 static nodes has GPS receiver. And a mobile node has no GPS receiver.

These graphs show the total of variance reduced as the mobile node moves. Espesially, we can see the improvement after the mobile node measures the relative distance to node 6 at time 2.

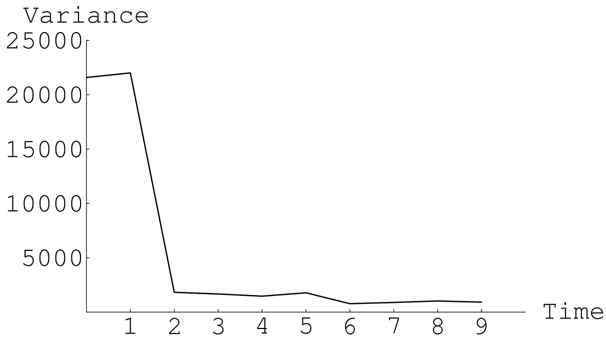


Fig. 7 Total of variance in 10 static nodes including 6 nodes equipping a GPS receiver.

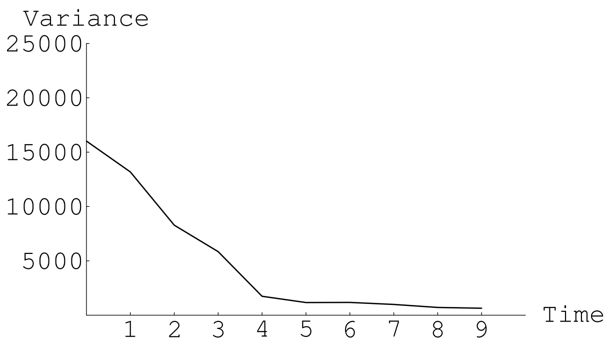


Fig. 8 Total of variance in 10 static nodes including 3 nodes equipping a GPS receiver.

These two environment of situation that all node has not less than one neighboring node that equipping the GPS receiver and that obtaines absolute position of that. And next, we shows the other simulation in the situation that 20 static nodes distributes that include 3 sensor node equipping the GPS receiver. Figure 9 shows the graphical model of static nodes. There are 4 nodes that has no neighbor node equipping the GPS receiver $(12, 15, 16, 20)$.

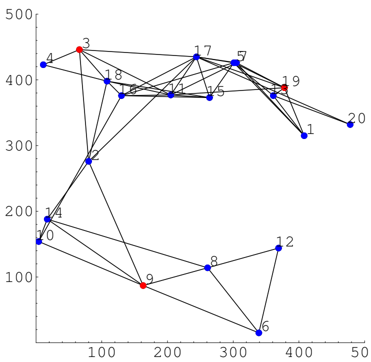


Fig. 9 The graphical model of 20 static nodes including 3 nodes equipping a GPS receiver.

Figure 10 shows each static node obtains the relative distance to mobile node.

And we shows the esimated position of 3 nodes that

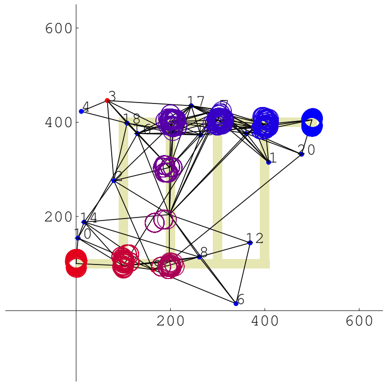
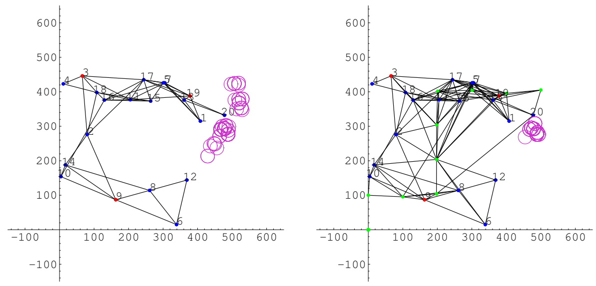


Fig. 10 The graphical model of 20 static nodes and the mobile node at each time

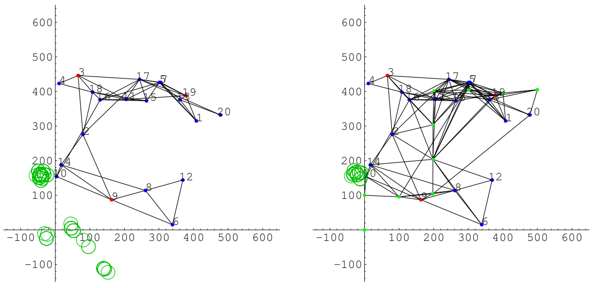
are node 20. Figure 11 shows estimated position of node 20. Node 20 has two neighboring nodes, which has no GPS receiver. Before the mobile node moves, particles that represent estimates are on part of circumference which center is node $17, 13$ as Figure11(A) shows. Through the distributed static nodes shown in Figure 11(A), one mobile node moves along a street shown in Figure 11(B), we assume that the mobile node moves as the same in previous simulation as shown in Figure 2. Figure 11(B) shows the estimated position of node 20 after the mobile node moves, and the ambiguity removed.



(A) (B)

Fig. 11 (A) shows the estimated position by the static node. (B) shows the estimated position of static node after the mobile node moves.

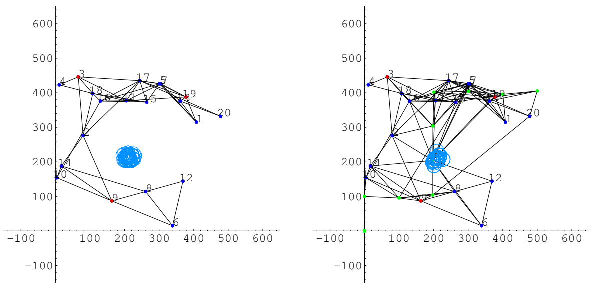
Node 12 also has no neighbor node that equipping GPS receiver. Figure 12(A) shows that the estimated position is on part of circumference but is not on real position. Figure 12(B) shows that the estimated position the node 12 after the mobile node moves is not nearer real position than estimation before the mobile node moves. And estimation in Figure 12 exists axisymmetric arrangement which axis is the line between the node 3 and 9 which equipping the GPS receiver. Some of node that has neighboring node equipping GPS receiver may estimate similally as node 12. We show this example in Figure 13. Figure 13 represent estimation of node 14. The result of estimation by static node shows that the estimated position of node 14 is not near real position of node 14 as shown in Figure 13(A). After the mobile node moves, the estimated position of node 14 is also near real position. The estimated position distributes exists axisymmetric ar-



(A) (B)

Fig. 12 (A) shows the estimated position od sensor node 12 by the static node. (B) shows the estimated position of sensor node 12 after the mobile node moves.

rangement which axis is the line between the node 3 and 9 which equipping the GPS receiver. And node 14 is measure the relative distance to mobile node at time 4, and obtain the measurement is 166.787. But comparing Figure 13 with Figure 16,the estimated positions of node 14 and mobile node at time 4 is almost the same.

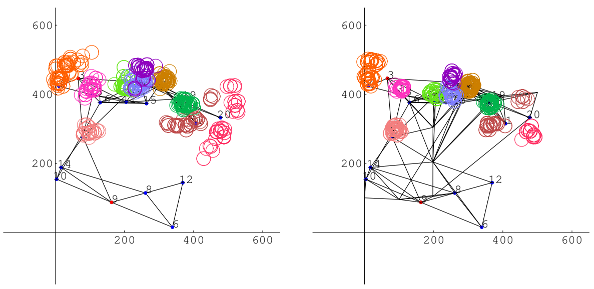


(A) (B)

Fig. 13 (A) shows the estimated position od sensor node 14 by the static node. (B) shows the estimated position of sensor node 14 after the mobile node moves.

These estimated position of node 12 and 14 is selected by weight that calculated in equaion 3, so once the position estimates not nearing real position that may be on part of circumference, the estimated position may not be able to improve.

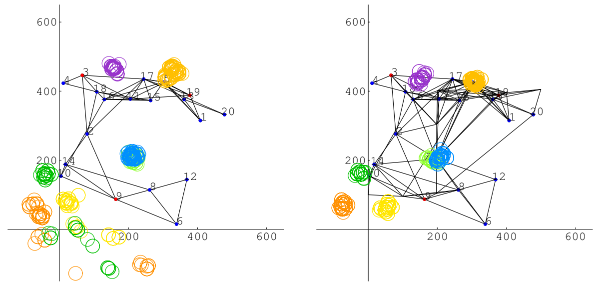
And we shows the estimation of other nodes. Figure 14 shows that estimation of node that estimated well, and these estimated position converged.



(A) (B)

Fig. 14 (A) shows the estimated position of node that estimated well by the static node that is not contain the node equipping GPS receiver. (B) shows the estimated position of static node after the mobile node moves.

Figure 15 shows that estimation of node that estimated position is not near real position, and these estimated position converged but this doesn't improve nearer the real position.



(A) (B)

Fig. 15 (A) shows the estimated position by the static node. (B) shows the estimated position of static node after the mobile node moves.

Figure 16 shows estimated position of mobile node at each time. The position of mobile node at each time is estimated well. The mobile node at each time measure relative distances to its neighboring node, which estimated position of oneself not correctly, but the estimated position of mobile node is well becased that estimated by using the information of moving distance.

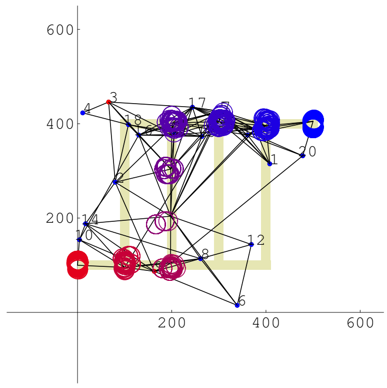


Fig. 16 The estimation of mobile node at each time.

Finally, we show graphs about variance of estimated position. Figure 17 show total variance of 20 static nodes, and we can see the totall variance reduce, and the estimated positions converged as the mobile node moves.

Figure 18 show the variance of node 20, and the estimated position of node 20 is the converged at the time 3 especially, because the node measure the relative distance to mobile node at time 3 firstly.

Figure 19 show the variance of node 12. The estimated position about node 12 in advance distributes on part of circumference as shown in Figure 12 and the node 12 measure the relative distance to mobile node at time 4 firstly. So, the variance is nonconstant before the relative distance obtained, but the variance is converged at the time 4 when the relative distance obtained.

Figure 20 show the variance of node 14. The estimated positon of node 14 is not near real positon of oneself as shown in Figure 13, but the distribution of estimaton is converged before mobile node moves, and the variance is not changed almostly ,so the information of mobile node doesn't influenced to node 14 partically.

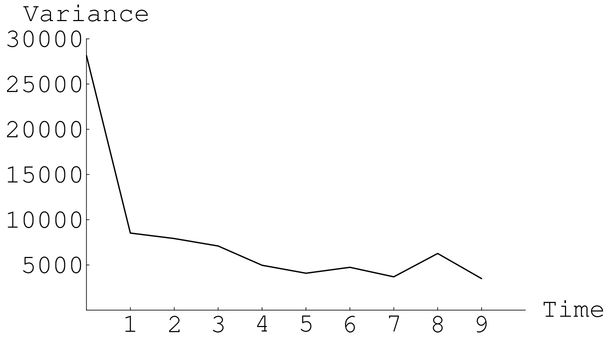


Fig. 17 Total variance of static nodes in simulation 3

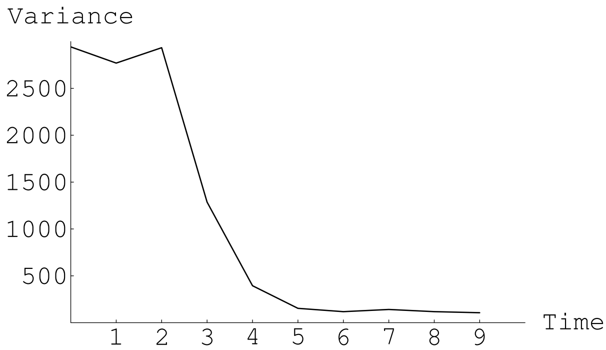


Fig. 18 The variance of node 20 at each time.

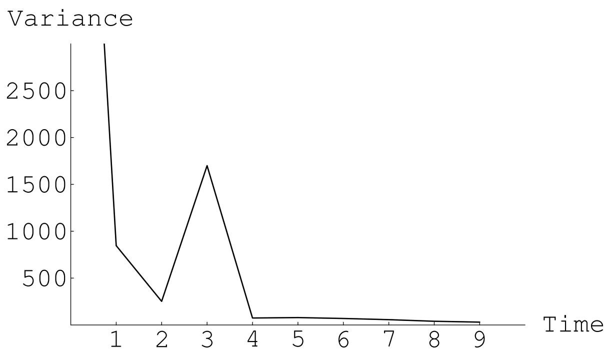


Fig. 19 The variance of node 12 at each time.

5. SUMMARY

In this article, we proposed a successive positioning method for networked sensors that including static node and a mobile one by means of the non-parametric belief propagation. Each sensor node can obtain relative distances to its neighboring nodes. Once distributed static nodes are positioned by using measurements of them, a mobile node moves through the distributed sensor nodes. If there are the ambiguity of the positions of some distributed nodes after estimation with only static nodes, by using the measurements obtained by a mobile sensor

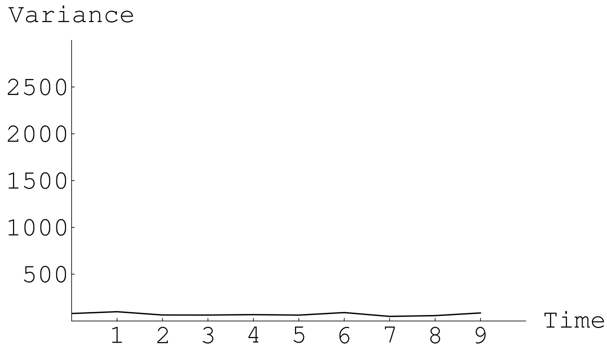


Fig. 20 The variance of node 14 at each time.

node, the proposed method improves the accuracy of the estimated positions of all nodes. In the situation that the number of node equipping GPS receiver is 3, some static node that has no GPS receiver cannot estimated near real position of oneself. Once the distribution of estimated position exists far from real position, it may not be able to improve estimation even if by using the information of mobile node.

REFERENCES

[1] A. T. Ihler, J. W. Fisher III, R. L. Moses, and A. S. Willsky, "Nonparametric Belief Propagation for Self-Localization of Sensor Networks," *IEEE J. Select Areas Commun.*, Vol. 23, No. 4, pp. 809-819, 2005.

[2] E. B. Sudderth, A. T. Ihler, W. T. Freeman, and A. S. Willsky, "Nonparametric Belief Propagation," *IEEE Computer Vision and Pattern Recognition.*, Vol. 1, pp. 605-612, 2003.